

Engineering Optimization

Engineering Optimization Theory and Practice

Fourth Edition

Singiresu S. Rao



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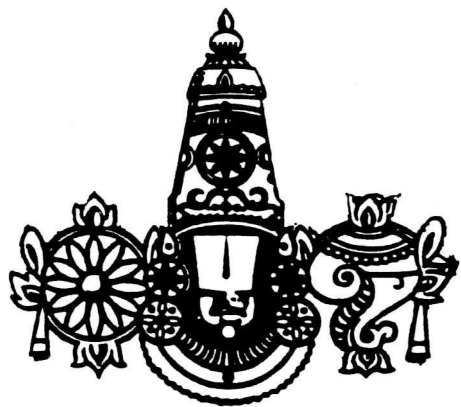
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Preface

The ever-increasing demand on engineers to lower production costs to withstand global competition has prompted engineers to look for rigorous methods of decision making, such as optimization methods, to design and produce products and systems both economically and efficiently. Optimization techniques, having reached a degree of maturity in recent years, are being used in a wide spectrum of industries, including aerospace, automotive, chemical, electrical, construction, and manufacturing industries. With rapidly advancing computer technology, computers are becoming more powerful, and correspondingly, the size and the complexity of the problems that can be solved using optimization techniques are also increasing. Optimization methods, coupled with modern tools of computer-aided design, are also being used to enhance the creative process of conceptual and detailed design of engineering systems.

The purpose of this textbook is to present the techniques and applications of engineering optimization in a comprehensive manner. The style of the prior editions has been retained, with the theory, computational aspects, and applications of engineering optimization presented with detailed explanations. As in previous editions, essential proofs and developments of the various techniques are given in a simple manner without sacrificing accuracy. New concepts are illustrated with the help of numerical examples. Although most engineering design problems can be solved using nonlinear programming techniques, there are a variety of engineering applications for which other optimization methods, such as linear, geometric, dynamic, integer, and stochastic programming techniques, are most suitable. The theory and applications of all these techniques are also presented in the book. Some of the recently developed methods of optimization, such as genetic algorithms, simulated annealing, particle swarm optimization, ant colony optimization, neural-network-based methods, and fuzzy optimization, are also discussed. Favorable reactions and encouragement from professors, students, and other users of the book have provided me with the impetus to prepare this fourth edition of the book. The following changes have been made from the previous edition:

- Some less-important sections were condensed or deleted.
- Some sections were rewritten for better clarity.
- Some sections were expanded.
- A new chapter on modern methods of optimization is added.
- Several examples to illustrate the use of Matlab for the solution of different types of optimization problems are given.

Features

Each topic in *Engineering Optimization: Theory and Practice* is self-contained, with all concepts explained fully and the derivations presented with complete details. The computational aspects are emphasized throughout with design examples and problems taken

from several fields of engineering to make the subject appealing to all branches of engineering. A large number of solved examples, review questions, problems, project-type problems, figures, and references are included to enhance the presentation of the material.

Specific features of the book include:

- More than 130 illustrative examples accompanying most topics.
- More than 480 references to the literature of engineering optimization theory and applications.
- More than 460 review questions to help students in reviewing and testing their understanding of the text material.
- More than 510 problems, with solutions to most problems in the instructor's manual.
- More than 10 examples to illustrate the use of Matlab for the numerical solution of optimization problems.
- Answers to review questions at the web site of the book, www.wiley.com/rao.

I used different parts of the book to teach optimum design and engineering optimization courses at the junior/senior level as well as first-year-graduate-level at Indian Institute of Technology, Kanpur, India; Purdue University, West Lafayette, Indiana; and University of Miami, Coral Gables, Florida. At University of Miami, I cover Chapters 1, 2, 3, 5, 6, and 7 and parts of Chapters 8, 10, 12, and 13 in a dual-level course entitled *Mechanical System Optimization*. In this course, a design project is also assigned to each student in which the student identifies, formulates, and solves a practical engineering problem of his/her interest by applying or modifying an optimization technique. This design project gives the student a feeling for ways that optimization methods work in practice. The book can also be used, with some supplementary material, for a second course on engineering optimization or optimum design or structural optimization. The relative simplicity with which the various topics are presented makes the book useful both to students and to practicing engineers for purposes of self-study. The book also serves as a reference source for different engineering optimization applications. Although the emphasis of the book is on engineering applications, it would also be useful to other areas, such as operations research and economics. A knowledge of matrix theory and differential calculus is assumed on the part of the reader.

Contents

The book consists of fourteen chapters and three appendixes. Chapter 1 provides an introduction to engineering optimization and optimum design and an overview of optimization methods. The concepts of design space, constraint surfaces, and contours of objective function are introduced here. In addition, the formulation of various types of optimization problems is illustrated through a variety of examples taken from various fields of engineering. Chapter 2 reviews the essentials of differential calculus useful in finding the maxima and minima of functions of several variables. The methods of constrained variation and Lagrange multipliers are presented for solving problems with equality constraints. The Kuhn–Tucker conditions for inequality-constrained problems are given along with a discussion of convex programming problems.

Chapters 3 and 4 deal with the solution of linear programming problems. The characteristics of a general linear programming problem and the development of the simplex method of solution are given in Chapter 3. Some advanced topics in linear programming, such as the revised simplex method, duality theory, the decomposition principle, and post-optimality analysis, are discussed in Chapter 4. The extension of linear programming to solve quadratic programming problems is also considered in Chapter 4.

Chapters 5–7 deal with the solution of nonlinear programming problems. In Chapter 5, numerical methods of finding the optimum solution of a function of a single variable are given. Chapter 6 deals with the methods of unconstrained optimization. The algorithms for various zeroth-, first-, and second-order techniques are discussed along with their computational aspects. Chapter 7 is concerned with the solution of nonlinear optimization problems in the presence of inequality and equality constraints. Both the direct and indirect methods of optimization are discussed. The methods presented in this chapter can be treated as the most general techniques for the solution of any optimization problem.

Chapter 8 presents the techniques of geometric programming. The solution techniques for problems of mixed inequality constraints and complementary geometric programming are also considered. In Chapter 9, computational procedures for solving discrete and continuous dynamic programming problems are presented. The problem of dimensionality is also discussed. Chapter 10 introduces integer programming and gives several algorithms for solving integer and discrete linear and nonlinear optimization problems. Chapter 11 reviews the basic probability theory and presents techniques of stochastic linear, nonlinear, and geometric programming. The theory and applications of calculus of variations, optimal control theory, and optimality criteria methods are discussed briefly in Chapter 12. Chapter 13 presents several modern methods of optimization including genetic algorithms, simulated annealing, particle swarm optimization, ant colony optimization, neural-network-based methods, and fuzzy system optimization. Several of the approximation techniques used to speed up the convergence of practical mechanical and structural optimization problems, as well as parallel computation and multiobjective optimization techniques are outlined in Chapter 14. Appendix A presents the definitions and properties of convex and concave functions. A brief discussion of the computational aspects and some of the commercial optimization programs is given in Appendix B. Finally, Appendix C presents a brief introduction to Matlab, optimization toolbox, and use of Matlab programs for the solution of optimization problems.

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