

Earthquake-induced hydrodynamic forces on reservoir roofs

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Abstract: The present paper describes the hydrodynamic loads on the roof of a water-filled reservoir or storage tank due to earthquake-induced sloshing. Initially, the paper summarizes available solutions for the water surface elevation in a rectangular reservoir subjected to harmonic and earthquake base motions, and as well an available formulation for the force on the roof of a rectangular reservoir. With this background, a new formulation for the force on the roof is developed, and selected results based on this are presented. A recommended design procedure is thereby proposed, and an example application is provided. The potential extension of the proposed formulation to other reservoir configurations is discussed. Although a validation of the proposed formulation based on laboratory test results is needed, it is suggested that in the interim the proposed formulation is adopted for design.

Key words: earthquakes, hydrodynamics, reservoirs, damping, impact forces, seismic design, waves.

Résumé : Cet article décrit les charges hydrodynamiques sur le toit d'un réservoir rempli d'eau ou d'un réservoir d'entreposage générées par ballottement de l'eau causé par un séisme. L'article commence par résumer les solutions disponibles pour l'élévation de la surface de l'eau dans un réservoir rectangulaire soumis à des mouvements harmoniques et sismiques de base; l'article traite également d'une formulation disponible pour la contrainte sur le toit d'un réservoir rectangulaire. Avec ces données de base, une nouvelle formulation de la contrainte sur le toit est développée et des résultats choisis, basés sur cette approche, sont présentés. Une procédure de conception recommandée est ainsi proposée et un exemple est fourni. L'extension potentielle de la formulation proposée à d'autres configurations de réservoir est abordée. Bien qu'une validation de la formulation proposée fondée sur les résultats d'essais en laboratoire soit requise, il est suggéré que la formulation proposée soit adoptée provisoirement pour la conception.

Mots-clés : séismes, hydrodynamique, réservoirs, amortissement ou atténuation, forces d'impact, conception sismique, vagues.

1. Introduction

The prediction of hydrodynamic loads and water surface elevations in water-filled reservoirs and tanks subjected to earthquake motions are important requirements in the seismic design of such structures. A simplified approach to estimating these for rectangular and circular reservoirs was presented by Housner (1957), and most design codes (ACI 2006; API 2007; ASCE 2006; AWWA 2007) are based on such an approach. This assumes that the force is made up of an "impulsive" fluid mass that accelerates in unison with the reservoir, and a "convective" fluid mass that undergoes resonant motions at the natural frequency of the lowest sloshing mode. Closed-form solutions of the corresponding linearized hydrodynamic problem for rectangular and circular tanks, taking account of specified levels of hydrodynamic damping, are also available (Isaacson and Subbiah 1991).

One particular aspect of the loading that may require par-

ticular consideration relates to the roof of a reservoir. If the water surface reaches the underside of the roof when the reservoir is subjected to seismic motions, then hydrodynamic forces will be exerted on the roof. To estimate such loads, a prediction of the water surface elevation (in the absence of a roof) is initially needed, and this needs to be applied to a suitable formulation for the force. An early approach to this problem is first summarized, and a modified approach is proposed in this paper. This paper then presents selected results based on this. Although a validation of the proposed formulation based on laboratory test results is needed, it is suggested that in the interim the proposed formulation be adopted in design. Thus, a recommended design procedure is summarized, and an example application is provided. The potential extension of the proposed formulation to other reservoir configurations is discussed.

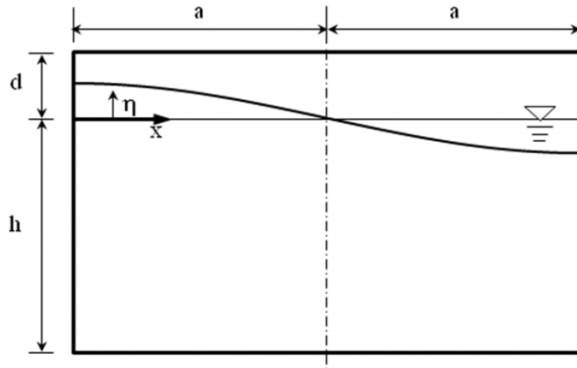
2. Estimate of surface elevation

Predictions of the water surface elevation are needed in the estimation of forces on roofs, and therefore these are initially summarized. Figure 1 provides a definition sketch for a rectangular reservoir: a denotes the half-length of the reservoir, h is the still water depth, and d is the elevation of the underside of the roof above the still water level. Also, w is the width of the reservoir. Initially, the case of a harmonic base motion is summarized, and this is subsequently extended to earthquake motions.

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Fig. 1. Definition sketch for a rectangular reservoir.

2.1. Harmonic motion

A closed-form solution for fluid sloshing in a rigid rectangular reservoir subjected to sinusoidal base motions is readily available (see, for example, Isaacson and Subbiah 1991). The solution is obtained on the basis of assumptions that the reservoir is rigid, the fluid is incompressible and inviscid, and the oscillation amplitude is small (such that the corresponding boundary value problem is linearized, and the amplitude does not exceed the freeboard). The solution provides a description of the corresponding fluid motion, and thereby provides an expression for the maximum water surface elevation. Although the solution was initially developed for the case of no-energy dissipation, it is possible to extend it to the case of energy dissipation corresponding to a specified damping coefficient, by assuming this occurs at the free surface (for example, Faltinsen 1978; Isaacson and Subbiah 1991).

The solution is expressed in terms of a set of eigenvalues, k_n , corresponding to each mode of sloshing. The eigenvalues, k_n , are obtained from successive roots to the equation $\cos(k_n a) = 0$. Thus, the values of $k_n a$ are given as

$$[1] \quad k_n a = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots \quad \text{for } n = 1, 2, 3, \dots$$

The natural frequencies of each sloshing mode, denoted ω_n , may be obtained in dimensionless form in terms of the k_n values from the equation

$$[2] \quad \frac{\omega_n^2 a}{g} = (k_n a) \tanh(k_n h)$$

where g is the gravitational constant.

The solution provides closed-form expressions for the water surface elevation, η , and other features of flow. The maximum value of η at the wall, which is denoted A , is of particular interest here and is given by

$$[3] \quad \frac{gA}{\omega U a} = \left| \left(1 + \frac{i\mu}{\omega} \right) \left\{ 1 - 2 \sum_{n=1}^{\infty} G_n(i\omega) \left[\frac{1}{(k_n a)^2} \right] \right\} \right|$$

where ω is the angular frequency of excitation, U is the velocity amplitude of the base motion, and $i = \sqrt{-1}$. Also, μ is a damping parameter accounting for energy dissipation within the fluid, and $G_n(i\omega)$ is a frequency-dependent function given by

$$[4] \quad G_n(i\omega) = \frac{\omega^2 + i\omega\mu}{\omega^2 + i\omega\mu - \omega_n^2}$$

The damping parameter, μ , may be expressed in terms of ζ_1 that is more commonly used, where ζ_1 denotes the fraction of critical damping for first-mode free oscillations that would correspond to the same level of damping. This is given as $\mu = 2\omega_1 \zeta_1$. The damping ratio may be estimated from an assessment of the energy dissipation mechanisms that are predominant (for example, Isaacson and Subbiah 1991). A damping ratio $\zeta_1 = 0.05$ is commonly adopted and appears to be reasonable for general application, unless more detailed information is available.

2.2. Modal response based on earthquake spectrum

In the case of reservoir motion due to an earthquake, the maximum response to the motion is generally described by a design spectral acceleration $S(T_n, \zeta)$ that corresponds to the maximum acceleration arising in a lightly damped single-degree-of-freedom system of natural period T_n and damping ratio ζ . Descriptions of S in a simplified form for use in structural design are available. For example, the 2005 *National building code of Canada* (NBCC) (NRCC 2005) provides such a description for many locations in Canada and for various soil conditions. It is noted that for longer periods (which are usually of primary interest with regard to sloshing) this code indicates that S should be taken as constant for $T_n > 4$ s, whereas S is then expected to decay as T_n^{-2} , as provided for in other codes such as ASCE-7-05 (ASCE 2006). Thus, the 2005 NBCC is expected to provide a notable overestimate of S for longer periods.

For such a motion, the maximum surface elevation, A_n , associated with the n th mode of sloshing may be derived from the closed-form solution for a harmonic motion (eq. [3]) and is given as

$$[5] \quad A_n = \left[\frac{2a}{(k_n a)^2} \right] S(T_n, \zeta)$$

where S is dimensionless with respect to g .

In the case when several modes need to be considered simultaneously, a common practice to estimate the overall maximum is based on taking the root of the sum of the squares of the maximum modal responses

$$[6] \quad A = \left[\sum_{n=1}^N A_n^2 \right]^{1/2}$$

where N is sufficiently large for convergence to occur. (In the analogous case of force predictions, there is an impulsive force component corresponding to $n = 0$, but this does not arise in the case of the surface elevation.) The significance of second and higher sloshing modes is often negligible, but in any case can be assessed by the use of eq. [6].

3. Force on roof

Now that estimates of the free surface elevation have been provided, these can be applied to obtain estimates of the resulting uplift force on the roof. An available formulation and a proposed formulation are described in turn.

3.1. SFPUC formulation

The San Francisco Public Utilities Commission (SFPUC 2003), based on Priestly (1986), summarizes an approach to estimating sloshing-induced hydrodynamic loads on the roof of a rectangular reservoir, and this has been used in seismic design. Figure 2a provides a definition sketch for this situation: d is the elevation of the underside of the roof above the still water level, A is the maximum free surface elevation above the still water level if the roof was absent, and $Y = A - d$ is the maximum surface elevation above the underside of the roof.

The pressure exerted on the roof is assumed to comprise an impact pressure p_i associated with the water impacting the roof, and a buoyancy component p_b associated with the ambient pressure at the roof's location if the surface elevation was unconstrained and able to rise above the roof. The recommended procedure is based on taking

$$[7] \quad p_i = \frac{1}{2} \rho V^2 C_s$$

$$[8] \quad p_b = \rho g Y$$

where ρ is the fluid density of the water, and V is the velocity of water at impact that is itself taken as

$$[9] \quad V = 2\pi A/T$$

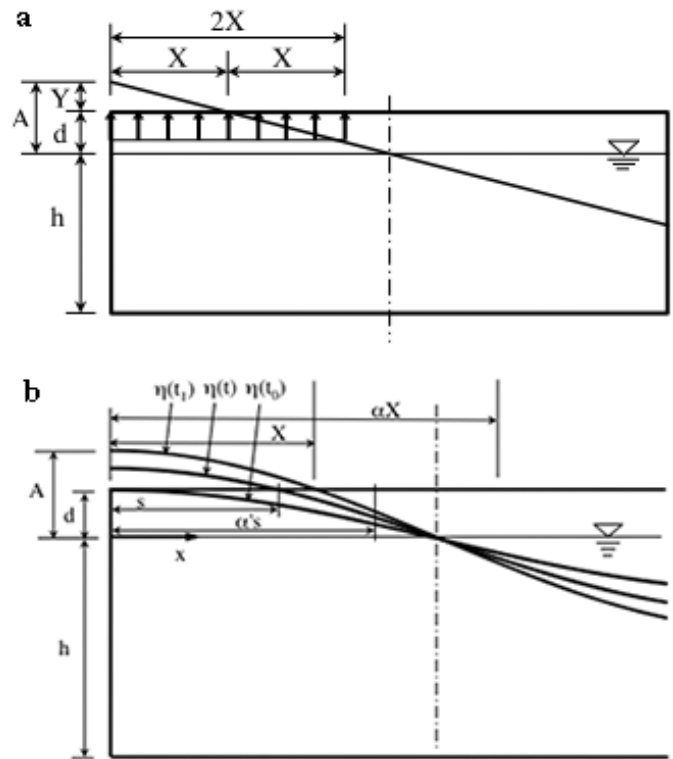
where T is the period of first-mode sloshing, and C_s is a wave impact coefficient that is stated to range up to 5.

In the SFPUC formulation, the water surface elevation is assumed to vary linearly between walls as indicated in Fig. 2a. X denotes the distance from the reservoir wall at which the unconstrained surface elevation coincides with the roof elevation, and thus $X = Ya/A$. The water that would otherwise lie above the roof must instead be displaced laterally, so that the water is actually in contact with the roof over a distance $2X$ rather than X from the wall. The combined pressure on the roof is assumed to be uniform and to extend a distance $2X$ from the reservoir wall. Based on the above equations, the force F on the roof is given as

$$[10] \quad F = 2Xw \left[\frac{1}{2} \rho V^2 C_s + \rho g Y \right]$$

Several improvements to the above formulation may be adopted. The assumed linear variation of water surface elevation from one reservoir wall to the other may notably underestimate the value of X , and instead a sinusoidal variation of η corresponding to first-mode sloshing is suggested. The impact force formulation that is used derives from the impact force acting on a horizontal circular cylinder of diameter $2X$, whereas a more suitable formulation corresponding to a horizontal roof may be possible. The buoyancy pressure exerted on the underside of the roof is not uniform but reduces to zero at a distance $2X$ from the wall. The impact force given by the above approach is independent of d , whereas it should reduce to zero as $d \rightarrow 0$ and as $d \rightarrow A$. Thus, the use of the unconstrained surface elevation amplitude A leads to an overestimate of the force in the case of a small clearance between the roof and the still water level, while the use of the maximum vertical velocity V leads to

Fig. 2. Definition sketch for the loading on the roof: (a) San Francisco Public Utilities Commission (SFPUC 2003) formulation; (b) proposed formulation.



an overestimate of the force in the case of a large clearance when the velocity of impact is much smaller. And finally, both the impact force and the buoyancy force are expected to vary with time through the passage of a wave, such that the overall force maximum may not be a sum of the component maxima. Based on these comments, a modification to the force formulation is developed below.

3.2. Proposed formulation

Figure 2b provides a definition sketch of the situation, with the unconstrained water surface profile now assumed to be sinusoidal, corresponding to first-mode sloshing. In the following, x is the horizontal coordinate measured from the wall, and time t is taken to be zero when the water surface is horizontal and moving upwards at the wall $x = 0$. The wave impact on the roof commences at time t_0 when the water surface just reaches the roof at the wall; then at time t_1 the unconstrained water surface crests at the wall and coincides with the roof elevation at some distance from the wall, $x = X$. At intermediate times, $t_0 \leq t \leq t_1$, the unconstrained surface elevation coincides with the roof elevation at a time-varying distance s from the wall. The corresponding wave profiles are shown in Fig. 2b.

3.2.1. Wave amplitude and wetted length

To analyze this case, expressions for the surface elevation η , the times t_0 and t_1 , the distance X , the time-varying vertical velocity v at the wall, and the time-varying distance s are needed. There are two features of the flow that need to be considered prior to a consideration of the force on the roof. One relates to a suitable choice of wave amplitude and the other to the wetted length of the roof.

With regard to the first, it has already been indicated that the use of the amplitude, A , is expected to lead to an overestimate of the force, because the incident flow at the time of impact does not, in fact, correspond to that of an unconstrained water surface. In particular, in the limiting case of zero roof clearance, there is no vertical motion of the fluid and so no vertical force exerted on the roof. Thus, for a very small roof clearance, the sloshing motion cannot build up so as to correspond to the amplitude, A , and the force will then be very small. Such features would not be predicted if the unconstrained amplitude, A , were to be used in the formulation.

Therefore, an expression for an equivalent amplitude, A_e , is developed. This can be estimated by considering the initial evolution of the unconstrained free-surface profile upon the onset of an earthquake motion. The free-surface elevation, η , may then be expressed in terms of the time history of the base acceleration, $\ddot{u}$, through Duhamel's integral. For first-mode sloshing with light damping, this may be approximated (for example, Isaacson and Subbiah 1991) as

$$[11] \quad \eta(t) = \left(\frac{8a}{\pi^2 g} \right) \int_0^t \ddot{u}(\tau) \exp[-i\zeta\omega(t-\tau)] \sin[\omega(t-\tau)] d\tau$$

The development of η for a base motion commencing from rest is needed, and can be obtained by taking the base acceleration to be given, for example, by

$$[12] \quad \ddot{u}(t) = \begin{cases} 0 & \text{for } t \leq 0 \\ F(t) \sin(\omega t) & \text{for } t > 0 \end{cases}$$

where $F(t)$ is a suitable ramping function that approaches 1 after a short time. It turns out that the resulting profile of η is not noticeably sensitive to $F(t)$. Equations [11] and [12] can be used to examine numerically the development of η over time, and, in particular, the growth of the amplitude in successive cycles. For a given clearance, d , the effective amplitude, A_e , may then be estimated by assuming that A_e corresponds to the amplitude that develops over a cycle beyond the instant that the amplitude first coincides with the given value of d . This imposes a limit to the growth in amplitude after initial water contact with the roof occurs. Such an approach has been used to examine numerically a relationship between A_e/A and d/A . It turns out that a reasonable fit to the numerical results is given by the simple approximation

$$[13] \quad \frac{A_e}{A} = \sin\left(\frac{\pi d}{2A}\right)$$

Now that the effective amplitude A_e has been determined, expressions for the various parameters identified above are given. First, the water surface elevation in the absence of the roof is taken as

$$[14] \quad \eta = A_e \cos(kx) \sin(\omega t)$$

where $k = \pi/2a$ is the wave number, $\omega = 2\pi/T$ is the wave angular frequency, and T is the wave period, all corresponding to first-mode sloshing. Then

$$[15] \quad \omega t_0 = \sin^{-1}(d')$$

$$[16] \quad \omega t_1 = \pi/2$$

$$[17] \quad kX = \cos^{-1}(d')$$

$$[18] \quad v = \omega A_e \cos(\omega t)$$

$$[19] \quad ks = \cos^{-1} \left[\frac{d'}{\sin(\omega t)} \right]$$

where $d' = d/A_e$.

The second feature of the flow to be considered relates to the wetted length of the roof. When the wave crests at the wall, $t = t_1$, the unconstrained surface elevation intersects the roof at a distance, X , from the wall (Fig. 2b). However, since the fluid does not rise above the roof, it is displaced laterally such that the water is then in contact with the roof over some greater distance, αX , rather than X . The SFPUC formulation is based on a linear variation of the unconstrained surface profile, leading to water contact over a distance $2X$, i.e., $\alpha = 2$. However, when the profile is taken to be sinusoidal, α may be different, but can nevertheless be calculated. Thus, the parameter α may be estimated by equating the volume of fluid that would lie above the roof to the volume of fluid displaced laterally as indicated in Fig. 2b. This eventually yields an expression for α as follows:

$$[20] \quad \frac{\sin(\alpha kX)}{\alpha kX} = d'$$

Since kX is a known function of d' , eq. [20] may readily be solved numerically for α , given a specified value of d' . In fact, as d' increases from 0 to 1, α reduces from 2 to $\sqrt{3}$.

Such an approach can readily be extended to assessing the varying wetted length at earlier times $t_0 \leq t \leq t_1$ prior to the wave cresting at the wall. Thus, at a general time t , the wetted length is taken to correspond to a distance $\alpha's$ rather than s , and α' can readily be obtained numerically as a function of time, using an analogous approach to that indicated above. In particular, at the instant of impact t_0 as $s \rightarrow 0$, it turns out that $\alpha' = \sqrt{3}$; and at time t_1 as $s \rightarrow X$, we have instead $\alpha' = \alpha$.

3.2.2. Impact and inertia forces

Now that various features of the flow have been considered, a formulation for the hydrodynamic force on the roof may be developed. This is done in a manner analogous to the development of the impact force on a horizontal circular cylinder. According to such an approach, the hydrodynamic force on the roof is given by the rate of change of fluid momentum arising from the deflection to the flow caused by the roof

$$[21] \quad F = \frac{d(mv)}{dt} = \frac{dm}{dt} v + m \frac{dv}{dt}$$

where m is the added mass of the fluid associated with the flow past the roof, and v is the fluid velocity normal to the roof if the roof were absent. The first term is associated with the rate of change of added mass with time and leads to the

impact or slamming force. The second term is an inertia force associated with the fluid acceleration.

The flow in the vicinity of the roof is rather complicated, and a relatively simple analogy to estimating the added mass is needed. Rather than considering the roof to correspond to a horizontal circular cylinder, we consider instead the flow near the roof to correspond to the two-dimensional potential flow normal to a flat plate of length $2b$. There are some differences between the actual flow around the roof and this reference flow (particularly with respect to the velocity at the ends of the plate), but overall the flow pattern is expected to be reasonably similar in the two cases, and also the pressure at the ends of the plate falls to zero as required.

For this reference case of a fully submerged plate in an infinite fluid due to a potential flow directed normal to its surface, the added mass per unit width on the plate is known and is given as $m' = \pi \rho b^2$ (for example, Sarpkaya and Isaacson 1981). However, in utilizing this analogy, it should be borne in mind that the flow adjacent to the roof corresponds to that past one half of the plate, and furthermore that a pressure is exerted only on the lower side of the plate. Therefore, the added mass per unit width in the present context is approximated as $m' = \pi \rho b^2/4$. By substituting the known expression for added mass into the first term on the right-hand side of eq. [21] and carrying out the time differentiation, an expression for the impact force as a function of time may be obtained

$$[22] \quad F_i = \frac{\pi}{2} \rho v b \dot{b} w$$

To apply the flat plate analogy to the case at hand, the plate half-length b is taken to correspond to $\alpha's$. Expressions for v and s have been given by eqs. [18] and [19], respectively, and a means of obtaining α' has been indicated. Taking account of the time differentiation that is needed, the force can thereby be obtained numerically as a function of time.

The maximum force is of particular interest and occurs at the instant of impact t_0 . An expression for the force then reduces to

$$[23] \quad \frac{F_i}{F_0} = 3 \left(\frac{A_e}{A} \right) \tanh \left[\left(\frac{\pi h}{2a} \right) \right] \left[\frac{1 - d'^2}{d'} \right]$$

where F_0 is a reference force defined as $F_0 = \rho g A a w$. The maximum impact force acts vertically at the wall, $x_F = 0$.

The inertia force, which corresponds to the second term on the right-hand side of eq. [21], is commonly used in ocean engineering applications and can readily be estimated. However, in the present context, the fluid acceleration $\dot{v}$ is always negative over the duration of interest ($t_0 < t < t_1$), and there is no flow above the roof. Therefore, as a conservative approximation, the inertia force is omitted from further consideration.

3.2.3. Buoyancy force

As in the SFPUC formulation, a "buoyancy" force associated with the ambient pressure within the fluid is also exerted on the roof. The buoyancy force would normally be estimated by taking it to be equal to the weight of water above the roof. This is a maximum when the wave crests at

the wall. However, the pressure falls to zero at $x = \alpha X$ rather than at $x = X$. Therefore, as a conservative approximation, it is assumed that the unconstrained water surface profile is stretched so as to lie above the roof over a distance αX rather than X . The maximum buoyancy force is thereby approximated as

$$[24] \quad F_b = \rho g \int_0^{\alpha X} \left[A_e \cos \left(\frac{kx}{\alpha} \right) - d \right] dx$$

This eventually leads to

$$[25] \quad \frac{F_b}{F_0} = \left(\frac{2\alpha}{\pi} \right) \left(\frac{A_e}{A} \right) [\sqrt{1 - d'^2} - d' \cos^{-1}(d')]$$

where F_0 is $\rho g A a w$ as before. The resultant force acts at a distance x_F from the wall given by

$$[26] \quad \frac{x_F}{a} = \frac{2\alpha}{\pi} \left[\frac{\cos^{-1}(d') \sqrt{1 - d'^2} - 1 + d' - \frac{1}{2} d' (\cos^{-1}(d'))^2}{\sqrt{1 - d'^2} - d' \cos^{-1}(d')} \right]$$

At earlier times, $t_0 \leq t \leq t_1$, the time-varying buoyancy force may be developed in an analogous manner, by utilizing $\alpha's$ in place of αX in the above development.

3.2.4. Total force

On the basis of the foregoing, the force is taken as the sum of the impact force and buoyancy force. However, it turns out that the maximum force corresponds either to the maximum impact force, occurring at time t_0 when the buoyancy force is zero, or to the maximum buoyancy force occurring at time t_1 when the impact force is zero. The condition whereby the overall maximum corresponds to one or the other may be obtained by equating the maximum impact force to the maximum buoyancy force. This eventually leads to an equation that expresses h/a as a function of d' . Thus, the maximum force corresponds to the impact force when

$$[27] \quad \frac{h}{a} \geq \frac{2}{\pi} \tanh^{-1} \left[\frac{2\alpha d'}{3\pi} \left(\frac{\sqrt{1 - d'^2} - d' \cos^{-1}(d')}{1 - d'^2} \right) \right]$$

and to the maximum buoyancy force otherwise.

4. Results and discussion

4.1. Surface elevation amplitude

For a harmonic motion, the dimensionless amplitude A given by eq. [3] is a function only of the frequency parameter $\omega^2 a/g$, the relative depth h/a , and the damping ratio ζ_1 , and is characterized by resonance peaks associated with the various modes of sloshing. However, the case of harmonic motion was presented primarily as a prelude to the case of an earthquake motion, and for an earthquake motion only first-mode sloshing is significant. Thus, results for a harmonic motion are only of peripheral interest here and so are not presented.

For an earthquake-induced motion, the amplitudes A_n at the various modes of sloshing and the maximum amplitude A (given by eqs. [5] and [6]) are dependent on a , h/a , and the acceleration spectrum, S , which itself may be specified

for a particular location and soil conditions. It is useful to examine the extent to which higher modes may affect the maximum A . Such an assessment has been carried out using the design spectral acceleration provided in the 2005 NBCC (NRCC 2005) for Vancouver, B.C. and for $\zeta_1 = 0.05$. Thus, Fig. 3 shows the percentage difference between the overall amplitude A and the first-mode amplitude A_1 as a function of the reservoir size, a , for various values of h/a . Figure 3 indicates that A_1 is generally very close to A , and underestimates A by less than 1% for a wide range of reservoir sizes, and in any case by less than 4% provided that $a \geq 5$ m. Therefore, the assumption that only first-mode sloshing is significant is quite reasonable. Even so, a modestly conservative assumption would be to use the overall amplitude, A , in place of the first-mode maximum, A_1 , in calculations of the force on the roof.

Housner (U.S. Atomic Energy Commission 1963) has provided an alternative expression for the maximum elevation in a rectangular reservoir, and it is of interest to compare this to the expression given here for the first-mode amplitude A_1 . Housner's expression is

$$[28] \quad A_H = \frac{0.527a}{\tanh[1.58(h/a)][(g/\omega_1^2 a \theta) - 1]}$$

where θ can be expressed as $\theta = S(T_1)$, $T_1 = 2\pi/\omega_1$, and ω_1 is given by eq. [2] with $n = 1$ and with $k_1 a$ taken as 1.58. Figure 4 shows the percentage difference between the above prediction, A_H , and the first-mode amplitude, A_1 , as a function of the reservoir size, a , for various values of h/a , and indicates how A_H overpredicts A_1 . It is noted that Housner's solution does not predict A to be proportional to base acceleration and fails for high values of θ ; whereas for low values of θ , it predicts the amplitude to be only 3% higher than that given by eq. [5]. Nevertheless, overall Housner's formula generally gives predictions that are notably higher than the closed-form solution for first-mode sloshing, and therefore it should not be used.

4.2. Maximum force on roof

First, the predicted variation of the force and force components with time is illustrated. Thus, Fig. 5 shows the time variation of the total force, the impact force, and the buoyancy force for the case $a = 50$ m, $h = 10$ m, $d = 2$ m for a spectral acceleration corresponding to the design spectrum for Vancouver, B.C. For this case, $A = 3.5$ m, so that $h/a = 0.2$ and $d/A = 0.58$. Figure 5 shows how the impact force falls from a maximum at the instant of initial impact to zero when the wave crests, whereas the buoyancy force increases from zero at the instant of initial impact to a maximum when the wave crests. In this particular case, the overall maximum force corresponds to the impact force at the instant of initial impact. Corresponding results for lower values of h/a would indicate that the overall maximum force would instead correspond to the maximum buoyancy force.

Figure 6 shows the dimensionless impact force maximum as a function of d/A . The influence of h/a is described in eq. [23] and is accounted for in the denominator of the dimensionless force that is used. Figure 6 indicates that the force falls to zero when $d/A \rightarrow 0$ and when $d/A \rightarrow 1$ as required, and is a maximum at an intermediate value, $d/A = 0.53$.

Fig. 3. Comparison of elevation maximum A and first-mode amplitude A_1 , shown as $A/A_1 - 1$ as a function of a for various values of h/a .

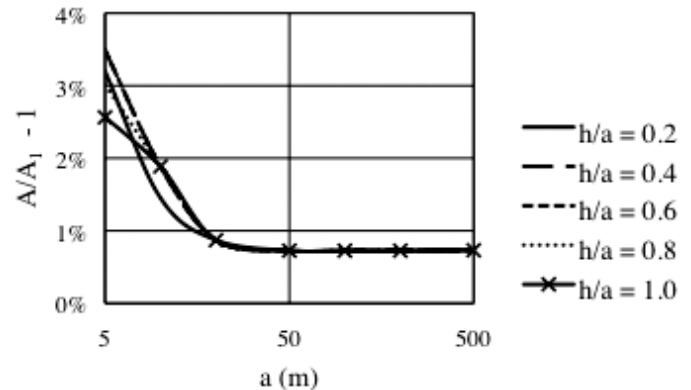


Fig. 4. Comparison of elevation amplitude A_H predicted by eq. [28] and first-mode amplitude A_1 , shown as $A_H/A_1 - 1$ as a function of a for various values of h/a .

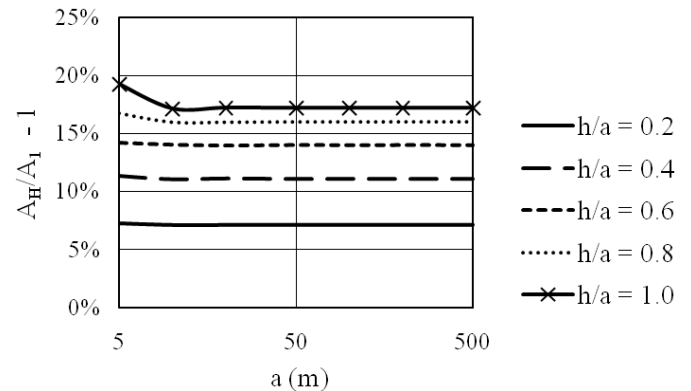


Fig. 5. Variation of force with time for $a = 30$ m, $h = 10$ m, $d = 1.2$ m ($h/a = 0.33$, $d/A = 0.58$). —, total force; - - -, impact force; ·····, buoyancy force.

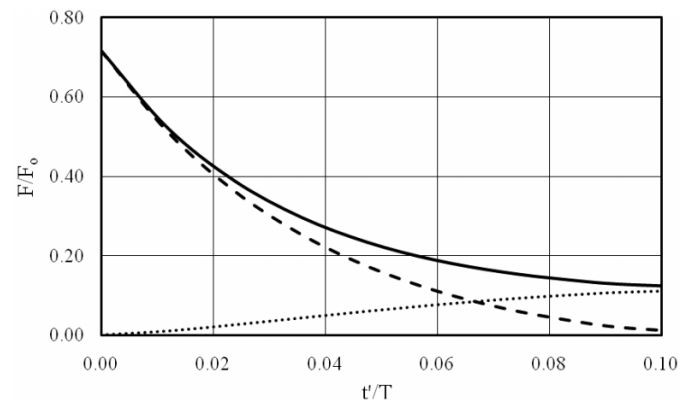
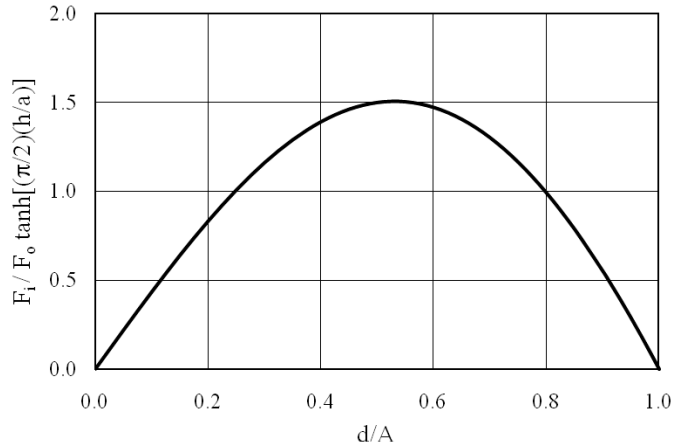
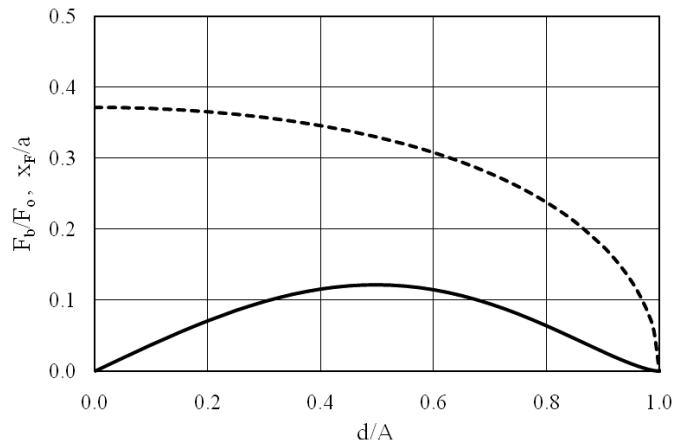


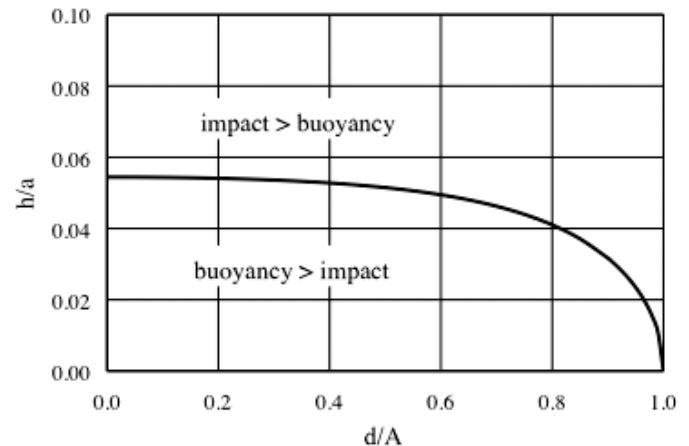
Figure 7 shows the dimensionless buoyancy force maximum and its location as a function of d/A . Again as expected, the buoyancy force maximum falls to zero when $d/A \rightarrow 0$ and when $d/A \rightarrow 1$, and is seen to reach a maximum with respect to d/A at $d/A = 0.50$. The maximum buoyancy force acts at a distance $x_F = 0.38a$ from the wall for smaller values of d/A , and is closer to the wall as d/A increases towards 1.

Fig. 6. Dimensionless impact force maximum as a function of d/A .**Fig. 7.** Dimensionless buoyancy force maximum and force location as functions of d/A . —, F_b/F_0 ; ---, x_F/a .

Finally, Fig. 8 shows a plot of eq. [27] and thus indicates the conditions whereby the overall force maximum corresponds either to the maximum impact force or to the maximum buoyancy force. Since the buoyancy force does not depend on h/a , whereas the impact force increases with h/a , the maximum force corresponds to the impact force at higher values of h/a . In particular, for $h/a \geq 0.054$, the maximum force corresponds to the maximum impact force regardless of the value of d/A .

4.3. Experimental validation

Given the various assumptions that have been made in developing the proposed formulation, it would clearly be prudent to validate or modify the methodology on the basis of experimental test results if at all possible. In an ocean engineering context, measurements involving wave impacts are known to be subject to considerable scatter, even under carefully controlled laboratory conditions (for example, Sarpkaya and Isaacson 1981). This is because the impact has a very short duration and is sensitive to slight changes in local conditions, for instance a slight slope of the impacted structure, dynamic effects, and so on. In any event, until such time that additional information becomes available, it is suggested that the current formulation provides a reasonably valid recommended procedure that improves the SFPUC formulation and so should supplant it.

Fig. 8. Comparison of impact and buoyancy force maxima, expressed on a plot of h/a as a function of d/A .

4.4. Alternative configurations

The case of a rectangular reservoir has been assumed in this paper, and it is of interest to examine the possibility of treating alternative reservoir configurations.

4.4.1. Sloping sides

In many applications, a reservoir may have sloping sides or floors. Isaacson and Ryu (1999) have suggested that such cases may be approximated as a rectangular reservoir, by utilizing either an equivalent water depth (for a gently sloping floor), or equivalent locations of the vertical sides (for steeply sloping sides). But apart from the selection of the equivalent depth or equivalent locations of sides, it is also possible that a surging flow over sloping sides may lead to higher wave amplitudes (in the absence of a roof) than would otherwise occur. To assess this, it is instructive to consider the analogous situation of ocean wave interactions with sloping seawalls. The Synolakis formula provides the runup on a seawall for a solitary wave approaching a slope (Li and Raichlen 2001), and may be used in the present context to estimate the relative increase in the wave amplitude at the wall. Thus, the formula may be recast to provide an estimate of the wave amplitude, A , as

$$[29] \quad \frac{A}{A_0} = 1.19 \sqrt{\cot(\beta)} \left(\frac{A_0}{h} \right)^{0.25}$$

where β is the slope angle, and A_0 is the wave amplitude for the case of a vertical wall. This formula fails at lower values of β , and thus it is suggested that the limit $A/A_0 \leq 1.5$ be imposed if necessary.

4.4.2. Circular reservoir

The case of a circular reservoir may be developed in an analogous manner to that presented for in this paper. The closed-form solution with energy dissipation has been given by Isaacson and Subbiah (1991). The eigenvalues k_n now correspond to successive roots of the equation $J_1'(k_n a) = 0$, where a now denotes the cylinder radius, J_1 is the Bessel function of the first kind and order one, and the prime denotes a derivative with respect to argument. The natural frequencies of each sloshing mode, ω_n , may be obtained in terms of the k_n values from eq. [2]. In this case, the force

formulation is more difficult, since the varying width, w , needs to be taken into account in the integrations of pressure to obtain the force. Because of this, the maximum force may no longer correspond to either the maximum impact or maximum buoyancy force, but needs to be evaluated numerically.

4.4.3. Irregular planform

When the reservoir has an irregular planform that cannot readily be approximated as circular or rectangular, the numerical method described by Isaacson and Ryu (1998) may be adopted to estimate the free surface elevation for application to the method given here. Their approach is based on an eigenfunction expansion of the velocity potential with respect to the vertical direction combined with a two-dimensional boundary element method with respect to the horizontal plane, with the planform of the reservoir discretized into a number of short segments. This would need to be combined with a numerical prediction of the force deriving from the present formulation. However, such an approach is cumbersome and should only be considered when the reservoir planform differs significantly from a rectangular or circular profile.

4.5. Recommended procedure

On the basis of the foregoing, the following procedure for estimating earthquake-induced hydrodynamic loads on the roof of a rectangular reservoir is recommended:

1. Results should be obtained for motions parallel to both pairs of sides to determine the more severe loading condition.
2. Specified values of a , h , d , w , and ρ are used, along with an assumed earthquake spectral acceleration $S(T_n, \zeta)$ for a given location and soil conditions. $S(T_n, \zeta)$ should be based on $\zeta = 0.05$, unless additional information relating to damping is available.
3. Equations [1] and [2] are used to obtain the eigenvalues k_n and resonant frequencies ω_n .
4. Equations [5] and [6] are used to obtain A_n and A , and hence the parameter d/A .
5. Equation [13] is used to obtain the effective amplitude A_e , and hence $d' = d/A_e$.
6. Equations [23], [25], and [26] are used to obtain the maximum force F and its location x_F .
7. The significance of reservoir configuration should be considered in a general way. If warranted, a more detailed analysis involving specialist expertise may then be required.

4.6. Example application

Using the recommended procedure outlined above, a sample set of calculations has been carried out for a rectangular reservoir that is 60 m long, 30 m wide, has a water depth of 10 m, a roof clearance of 1.2 m, and is assumed to be located in Vancouver, B.C., under site class C conditions (very dense soil and soft rock). That is $a = 30$ m, $w = 30$ m, $h = 10$ m, and $d = 1.2$ m. A damping level $\zeta = 0.05$ is assumed, and the water density is taken as $\rho = 1000$ kg/m³.

For such conditions, eqs. [1] and [2] indicate that first-mode sloshing occurs at $\omega_1 = 0.50$ rad/s ($T_1 = 12.6$ s). The next step is to select a suitable design spectral acceleration S ; the value specified by the 2005 NBCC is used for this

purpose. It has already been noted that this code may provide a notable overestimate of S for longer periods, but nevertheless it is still used here for illustrative purposes. For the specified conditions, the code provides $S = 0.085g$ at T_1 .

Equations [5] and [6] indicate that the first-mode sloshing amplitude and the overall sloshing amplitude are $A_1 = 2.07$ m and $A = 2.08$ m, so that $d/A = 0.58$. From eq. [13], the effective sloshing amplitude is then estimated to be $A_e = 1.64$ m, and therefore $d' = 0.73$.

Equations [23] and [25] indicate that the maximum inertia and buoyancy forces are given by 13.2 and 2.2 MN, respectively. Therefore, the maximum force on the roof is taken to be 13.2 MN, and it acts close to the wall, $x_F = 0$ m.

Considering instead an earthquake motion parallel to the shorter pair of sides such that $a = 15$ m and $w = 60$ m, the overall sloshing amplitude is calculated to be $A = 1.05$ m. Since $d = 1.2$ m, there is no water contact with the roof and therefore no force exerted on the roof. Therefore, overall, the maximum force on the roof is taken to be 13.2 MN, and to act close to the wall, $x_F = 0$ m.

5. Summary and conclusions

The present paper proposes a methodology for estimating hydrodynamic loads on the roof of a water-filled reservoir or storage tank due to earthquake-induced liquid sloshing. Initially, the paper summarizes available solutions for the water surface elevation in a rectangular reservoir subjected to harmonic and earthquake base motion, and as well an available formulation for the force on the roof of a rectangular reservoir. With this background, a new formulation for the force on the roof is developed, and selected results based on this are presented. This entails a consideration of impact and buoyancy force components. The former is evaluated by considering an analogy with the flow past a flat plate. It is found that the maximum force corresponds either to the maximum impact force or to the maximum buoyancy force, and so is relatively straightforward to estimate. A recommended design procedure is thereby proposed, and an example application is provided. The potential extension of the proposed formulation to other reservoir configurations is discussed. Given the assumptions that are made, a validation of the proposed formulation based on laboratory test results is needed, but it is suggested that, in the interim, the proposed formulation is an improvement on the approach that is currently used and so should be adopted for design. It is found that an earlier formula (based on eq. [28]) for estimating the maximum sloshing elevation is unreliable and should not be used.

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- d elevation of underside of roof above still water level
- d' relative roof elevation, $d' = d/A_e$
- F total force
- F_b buoyancy force
- F_i impact force
- F_0 reference force, $F_0 = \rho g A a w$
- g gravitational constant
- $G_n(i\omega)$ frequency dependent function, see eq. [4]
- h still water depth
- i $i = \sqrt{-1}$
- $J'_1(k_n a)$ derivative of J_1 with respect to argument
- J_1 Bessel function of first kind and order 1
- k wave number of first-mode sloshing
- k_n eigenvalues corresponding to each mode of sloshing
- m added mass of fluid
- m' added mass per unit width
- p_i impact pressure
- p_b buoyancy pressure
- s distance from wall of intersection of unconstrained surface elevation with roof, see Fig. 2b
- S spectral acceleration
- t time
- t_0 time at instant of initial impact
- t_1 time at which wave crests at the wall
- T wave period of first-mode sloshing
- T_n natural period, $= 2\pi/\Omega_n$
- u base velocity
- U amplitude of base velocity
- $\dot{u}$ base acceleration
- v vertical fluid velocity
- $\dot{v}$ vertical fluid acceleration
- V amplitude of vertical fluid velocity
- w width of rectangular reservoir
- x horizontal coordinate, measured from reservoir wall
- X value of s at time t_1 when wave crests at the wall, see Fig. 2b
- x_F x coordinate of resultant force
- Y elevation of unconstrained wave crest above roof, $A - d$
- α αX : maximum length of water contact with the roof, see Fig. 2b
- α' $\alpha's$: length of water contact with the roof at time t , see Fig. 2b
- β angle of sloping wall above horizontal
- ζ damping ratio
- ζ_1 fraction of critical damping for first-mode free oscillations
- η free surface elevation
- θ $\theta = S(T_1)$
- μ damping parameter
- ρ fluid density
- ω angular frequency
- ω_n natural frequency, see eq. [2]

List of symbols

- a half length of rectangular reservoir
- A amplitude of surface elevation in absence of roof
- A_0 wave amplitude for a vertical wall
- A_e effective amplitude of amplitude of surface elevation
- A_H amplitude of surface elevation based on Housner (1957), see eq. [28]
- A_n amplitude of surface elevation of n th sloshing mode
- b plate half-length
- $\dot{b}$ time rate of change of b
- C_s wave impact coefficient